Statics & Dynamics in Biomechanics: Joints

References


Newton’s Laws (Sir I. Newton d. 1727)

• The foundations of mechanics
• Can be used to answer problems in biomechanics where parts of the body can be approximated as rigid elements, e.g.
  – the static equilibrium of joints
  – mechanics of gait
  – dynamics of running, jumping and sports
  – hydrostatics of biofluids such as blood
  – equilibrium of pressurized membranes

Newton’s Laws

• Newton’s First Law
  \[ f = 0 \quad \Rightarrow \quad v = \text{constant} \]

• Newton’s Second Law
  inertial force (ma) balances the external force, f
  \[ \text{If } f = 0 \quad \frac{d}{dt}(mv) = f \]
  \[ \Rightarrow \quad f - ma = 0 \]

• Newton’s Third Law
  For a system of particles \( I = 1, 2, \ldots, N \)
  \[ f_i + f_{ji} = 0 \]
  where \( f_{ij} = \text{force of interaction of particle } I \text{ on particle } J \)
**Equations of Linear Motion**

- The total force acting on particle I is the external force plus the sum of the interaction forces:
  \[ f_i = f_i^{(e)} + \sum_{j=1}^{N} f_{ij} \]
- From Newton’s Second Law:
  \[ f_i = \frac{d}{dt}(m\nu_i) \]
- Hence, for \( I = 1, 2, 3, \ldots N \):
  \[ \frac{d}{dt}(m\nu_I) = f_i^{(e)} + \sum_{j=1}^{N} f_{ij} \]

(A system on N ordinary differential equations)

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**Static Equilibrium**

- When all N particles in the system have zero acceleration:
  \[ f_i^{(e)} + \sum_{j=1}^{N} f_{ij} = 0 \]
- Sum over all N particles:
  \[ \sum_{i=1}^{N} f_i^{(e)} + \sum_{i=1}^{N} \sum_{j=1}^{N} f_{ij} = 0 \]
- By Newton’s Third Law, the second term = 0, hence
  \[ \sum_{i=1}^{N} f_i^{(e)} = 0 \]

For a body in equilibrium, the sum of all external forces acting on the body is zero

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**Couples and Moments**

- A *couple* is a pair of forces, \( f \) and \( -f \), equal in magnitude, opposite in direction, and separated by a distance, \( d \). It can cause a *rotation*
- The *moment* of a force \( f \) about a point \( O \) is the *vector product* of \( f \) and the position vector, \( r \), from \( O \) to the point of action of the force vector \( M = r \times f \)
- Newton’s laws also hold for angular velocity, moment, and angular momentum
Newton’s Angular Laws

• Newton’s First Law (angular)
  \[ \mathbf{M} = 0 \implies \omega = \text{constant} \]

• Newton’s Second Law (angular)
  Moment of momentum balances the external moment
  \[ \text{if } \mathbf{M} = 0 \quad \frac{d}{dt}(I \times \omega) = \mathbf{M} \]
  \[ \implies \mathbf{M} - I \times \alpha = 0 \]

• Newton’s Third Law (angular)
  For a system of particles \( I = 1, 2, \ldots, N \)
  \[ \mathbf{M}_I + \mathbf{M}_J = 0 \]
  where \( \mathbf{M}_{ij} \) = moment of interaction of particle I on particle J

Angular Momentum, \( I \alpha \)

• \( \omega \) is the angular velocity
• \( \alpha \) is the angular acceleration
• \( I \) is the moment of inertia
• For a rigid system of particles \( I = 1, 2, \ldots, N \), each with mass \( m_i \) and radius \( r_i \) from a fixed origin e.g. the center mass:
  \[ I = \sum_i m_i r_i \]
  \[ \omega_i = \dot{r}_i \]
  \[ \alpha_i = \ddot{r}_i \]
  Therefore, if \( \mathbf{M} = 0 \quad \mathbf{M} - I \times \alpha = 0 \]
  \[ \implies \frac{d}{dt} \left( \sum_i m_i r_i \times r_i \right) = \sum_i r_i \times \ddot{r}_i^{(i)} \]

Equations of Angular Motion

• The total moment acting on particle I is the external force plus the sum of the interaction moments:
  \[ \mathbf{M}_I = \mathbf{M}_I^{(e)} + \sum_{J \neq I} \mathbf{M}_{ij} \]

• From Newton’s Second Law:
  \[ \mathbf{M} = \frac{d}{dt} (I \times \omega) \]

• Hence, for \( I = 1, 2, 3, \ldots, N \):
  \[ \frac{d}{dt} (I \times \omega_I) = \mathbf{M}_I^{(e)} + \sum_{J \neq I} \mathbf{M}_{ij} \]
  (A system on N ordinary differential equations)
Moment Equilibrium

- If a body is in equilibrium, there is no tendency to rotate and \( \mathbf{M} = 0 \), so:
  \[
  \sum_{i=1}^{N} M^{(e)}_i + \sum_{j=1}^{N} M_{ij} = 0
  \]
  
- Sum over all N particles:
  \[
  \sum_{i=1}^{N} M^{(e)}_i + \sum_{i=1}^{N} \sum_{j=1}^{N} M_{ij} = 0
  \]
  
- By Newton’s Third Law, the second term = 0, hence
  \[
  \sum_{i=1}^{N} M^{(e)}_i = \sum_{i=1}^{N} \mathbf{r}_i \times \mathbf{t}^{(e)}_i = 0
  \]
  
For a body in equilibrium, the sum of all external moments about any fixed point is zero.

Statics in Biomechanics

Typical problems:
- Forces in muscles and ligaments?
- Loads on joints?
- How is load distributed at a joint?
- What tension in the wall of a blood vessel or the heart balances the pressure force exerted by the blood?
- How does venous blood pressure affect by standing up from supine?

Skeletal Joints

- Human body is rigid but flexible – flexibility is due to joints or articulations
- Joints provide mobility and stability in varying degrees
- Degree of mobility vs. stability differs:
  - Synarthrodial joints – tight fitting, no mobility, e.g skull
  - Amphiarthrodial joints – slight motion, intervening cartilaginous or ligamentous tissue, e.g. vertebrae
  - Diarthrodial joints – articulating, most mobility, e.g.
    - shoulder (ball and socket), high mobility (triaxial) but reduced stability and increased vulnerability;
    - elbow, less mobility (biaxial) and less prone to injury
- Joint mobility depends on nature of surfaces, structure of capsular ligament, length of ligaments, arrangement of muscles.
Diarthrodial joints

1. bone
2. ligamentous capsule
3. synovial membrane (internal lining of ligamentous capsule)
4. synovial fluid (lubricates and damps)
5. articular cartilage (provide bearing surfaces, absorb load, damp impacts)
6. articular cavity

Diarthrodial joints types:

• hinge (elbow/ankle)
• pivot (radioulnar)
• condyloid (wrist)
• saddle (carpometacarpal of thumb)
• ball and socket (shoulder/hip)
• gliding (vertebral facets)

Skeletal Muscles

• three types of muscles: skeletal, smooth and cardiac
  • striated due to organization of contractile filaments as opposed to unstriated (smooth) muscle
  • voluntary rather than involuntary
  • contraction - the ability of muscles to shorten or develop tension
  • contraction types
    – concentric (shortening)
    – isometric (static)
    – eccentric (lengthening)
• agonist muscles (cause a motion)
• antagonist muscles (oppose a motion)
Questions in Joint Statics

- What tension must the neck extensor muscles exert on the head to support it in a specified position?
- When a person bends, what would be the force exerted by the erector spinae on the fifth lumbar vertebra?
- How does the compression at the elbow, knee, and ankle joints vary with externally applied forces and with different segmental arrangements?
- How does the force on the femoral head vary with loads carried in the hand?
- What are the forces involved in various muscle groups and joints during different exercise conditions?

Basic Assumptions of Joint Statics

- The vector characteristics of the system are known (requires anthropometric data)
  - anatomical axes of joint rotation
  - muscle attachments
  - muscle lines of action
  - segment weights and centers of mass
- The system can be simplified to be statically determinate:
  - ignore dynamics, i.e. no inertial forces
  - ignore deformability of muscles, tendons and bones (rigid motions only)
  - only one muscle or muscle group acts
  - no joint friction

Static Equilibrium of Mechanical Systems

**Statically Determinate System**
- The number of unknown forces (N) equals the number of equations (M) from the force and moment balances of statics

**Indeterminate System (overdetermined)**
- Real joints have too many muscles (N>M)
- Additional information is needed to find an answer from infinitely many possibilities

**Unstable System (underdetermined)**
- Not enough forces for the system to be static (N<M)
- It becomes an unstable mechanism
Example: Pin-Jointed Trusses

The Elbow

Three major bones at the elbow:
- humerus (1)
- radius (4)
- ulna (5)

Three separate articulations:
- The humeroulnar joint is a hinge; it allows only uniaxial rotations, flexion or extension.
- The humeroradial joint is also a hinge joint.
- The proximal radioulnar joint is a pivot joint; the radius and ulna to undergo relative rotation about the longitudinal axis of one or the other bone, giving rise to pronation (palm-up to the palm-down) supination, (palm-down to the palm-up).

Elbow Joint Muscles

Biceps Brachii
- the most powerful elbow flexor attached to radius, coracoid process and glenoid fossa

Brachialis
- important flexor
- attachments at humerus and ulna

Triceps Brachii
- major extensor
- attachments to scapula, humerus, ulna

Pronator Teres and Supinator
- pronation and supination
- attached to the humerus, ulna, and radius
Forces on the Lower Arm

- O is axis of elbow rotation
- A is the attachment of the biceps on the radius (OA=a)
- B is the center of gravity of the forearm (OB=b) which has weight W
- C is the center of mass on the hand (OC=c) of the weight W₀
- F_M is the force exerted by the biceps
- F_J is the reaction force at the elbow joint

Forces on the Lower Arm: Solution

Determine the magnitudes of the muscle tension and the joint reaction force at the elbow:
- The x force balance is satisfied trivially
- The y force balance and the balance of the moments about O are sufficient to solve for the unknown muscle and joint forces:
- Given a = 4 cm; b = 15 cm; c = 35 cm; W = 20N; W₀ = 80N
- We get:
  - Muscle force, F_M = 775N
  - Joint force, F_J = -675N

Forces on the Lower Arm: Remarks

- Muscle force is about 10x > the weight of the object
- Thus, during lifting, it is disadvantageous to have a muscle attachment close to the elbow joint
- But the closer the muscle is to the joint, the larger the range of motion and the faster the hand can move.
- When the forearm is flexed at 90° to the line of action of F_M, the muscle tension has only a normal rotational component
- For other flexed positions of the forearm, the muscle force also has a tangential component:
  - it has a stabilizing effect when flexion angle >90°
  - and a dislocating role when the flexion angle <90°
- Synovial fluid is to distribute the forces at the joint over a relatively large surface area as pressures
- To make the analysis statically determinate, we disregarded other elbow flexors, e.g. brachialis and brachioradialis
**Topic 2: Summary of Key Points**

- *Newton’s Laws* can be used for many biomechanical analyses where the parts of the body can be considered rigid.
- For a body in *static equilibrium*, the sum of all the external forces and moments is zero.
- *Joints* provide flexibility to varying degrees.
- *Skeletal muscles* are voluntary and striated; they come in agonist-antagonist pairs.
- The usual *assumptions of joint statics* are designed to make the problem *statically determinate*.
- The tangential component of muscle force at a joint can be *stabilizing or dislocating* depending on joint flexion angle.