Muscle Mechanics

- Twitch
- Tetanus
- Isometric contraction
- Isotonic contraction
- Concentric contraction
- Eccentric contraction
- Transient loading

The Sarcomere

Anisotropic to polarized light

~ 2.0 μm

Muscle Contraction

We know that muscle shortening corresponds to the sliding of thin (actin) filaments past thick (myosin) filaments.

Most widely accepted mechanism for generation of force is the formation of connections between these filaments by crossbridges.
Tests of Contractile Mechanics

- Isometric - constant length
- Isotonic - constant force
- Transient

Fast vs. Slow Twitch Muscle

Duration of twitch corresponds to functional requirements of muscle

- **Fast twitch muscle**
  - White muscle
  - Lower concentrations of RBCs and myoglobin, e.g. ocular muscle

- **Slow twitch muscle**
  - Red muscle
  - E.g. soleus muscle

Isometric Twitch Tension $T$

- Ocular (eye)
- Gastrocnemius (calf)
- Soleus (calf)

Twitch and Tetanus

- When a series of stimuli is given, isometric muscle force rises to a plateau (unfused tetanus) which ripples at the stimulus frequency. As stimulus frequency is increased, the plateau rises and becomes a smooth fused tetanus.
Isometric Length-Tension Relationships

- Tension-length curves for frog sartorius muscle at 0°C
- Fixed length
- Electrical Stimulation
- Measure Force Generation

Isometric Tension: Sliding Filament Theory

- Developed tension versus length for a single fiber of frog semitendinosus muscle
- Ascending limb is dependent on Ca²⁺ concentration
- Results in Isometric Length-Tension Relationship of muscle

Isotonic Shortening: Force Velocity Relations

- (a) Quick-release isotonic apparatus
- (b) Isometric tetanic stimulation followed by quick release and isotonic shortening by Δx₁ and Δx₂
The shortening part ($V>0$) of the curve was computed from Hill's equation with $c = 4$.

The asymptotes for Hill's hyperbola (broken lines) are parallel to the $T/T_0$ and $V/V_{max}$ axes.

Mechanical power output is the product of $T$ and $V$.

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Hill's Equation

Original Form:

\[(T+a)(V+b)=b(T_0+a)\]

Dimensionless forms:

\[\frac{V}{V_{max}} = 1 - \frac{T}{T_0} + c\left(\frac{T}{T_0}\right)\]

\[\frac{T}{T_0} = 1 - \frac{1}{1 + c\left(\frac{V}{V_{max}}\right)}\]

$a, b =$ asymptotes

$T_0 =$ Isometric force

$V_{max} =$ velocity at $T = 0$

$c = T_0/a$

(ranges from 1.2-4.0)

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Hill's Three Element Model

**Fundamental Assumptions:**

- Resting length-tension relation is governed by an elastic element in parallel with a contractile element. In other words, active and passive tensions add. The parallel elastic element is the passive properties.

- Active contractile element is determined by active length-tension and velocity-tension relationships only.

- Series elastic element becomes evident in quick-release experiments.

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Fig 9.8.1 from text. Hill’s functional model of muscle
Hill’s Three-Element Model (basic equations)

\[ T = T^{(p)} + T^{(s)} = P(L) + S(\eta, \Delta) \]

- stress contributed by the parallel elastic element
  \[ T^{(p)} = P(L) \]
- stress contributed by the activated actin–myosin filaments:
  \[ T^{(s)} = S(\eta, \Delta) \]

\[ S(\eta, \Delta) \] is identically zero when the muscle is at rest; so that
\[ S(\eta, \Delta) = 0 \quad \text{when} \quad \eta > 0 \]
and
\[ S(\eta, \Delta) = 0 \quad \text{when} \quad \eta = 0 \]

Example analysis of Hill’s model

Write \( T \) in terms of parallel + series terms

\[ T = T^{(p)} + T^{(s)} = P(L) + S(\eta, \Delta) \]

where \( P(L) = 0 \quad S(\eta, \Delta) = 0 \)

\[ \frac{dT}{dt} = \frac{\partial P}{\partial L} \frac{dL}{dt} + \frac{\partial S}{\partial \eta} \frac{d\eta}{dt} + \frac{\partial S}{\partial \Delta} \frac{d\Delta}{dt} \]

\[ \frac{dT}{dt} = \frac{\partial P}{\partial L} \frac{dL}{dt} + \left( \frac{\partial S}{\partial \eta} \frac{d\eta}{dt} + \frac{\partial S}{\partial \Delta} \frac{d\Delta}{dt} \right) \frac{d\eta}{dt} \]

Special Cases of Hill’s Model

Isometric contraction: \( L = \text{constant} \) and \( \frac{dL}{dt} = 0 \),

\[ \frac{dT}{dt} = \left( \frac{\partial S}{\partial \eta} + \frac{\partial S}{\partial \Delta} \right) \frac{d\Delta}{dt} \]

Isotonic contraction: \( T = \text{constant} \) and \( \frac{dT}{dt} = 0 \),

\[ \left( \frac{dP}{dL} + \frac{\partial S}{\partial \eta} \right) \frac{dL}{dt} + \left( \frac{\partial S}{\partial \eta} + \frac{\partial S}{\partial \Delta} \right) \frac{d\eta}{dt} \frac{d\Delta}{dt} = 0 \]

Need third experiment to identify Series Elastic Element. Use Isometric-Isotonic changeover experiment.
Cross-Over Experiment

Length

Tension

Isometric  Isotonic

Isometric contraction:

Isotonic contraction:

Limitations of Hill Model
Division of forces between parallel and series elements and division of displacements between contractile and elastic elements is arbitrary (i.e. division is not unique). Structural elements cannot be identified for each component.

Hill model is only valid for steady-shortening of tetanized muscle.

1) For a twitch we must include the time-course of activation and hence define "active state"

2) Transient responses observed not reproduced

Series elasticity is not observed. Properties of tendon and crossbridge itself

Small Length Step Response

Tetanized single frog muscle fiber at 0°C during a 1% shortening step lasting 1 ms
Instantaneous and Plateau Tension

Solid lines: sarcomere length = 2.2 μm (near maximal myofilament overlap). Broken lines: sarcomere length = 3.1 μm (39% myofilament overlap). Thus instantaneous tension $T_1$ reflects crossbridge stiffness and number of attached crossbridges which varies with myofilament overlap.

Huxley and Simmons Model (1971)

Two stable attached states of S-1 head. Thin filament displacement $y$ stretches S-2 spring generating force.

Calculated curves of $T_1$ and $T_2$ versus length step $y$ showing predictions of Huxley and Simmons model.

Both Elastic Elements are Inside the Contractile Element

Actin

Myosin

Titin

Z-line

$T$
Muscle Mechanics: Summary of Key Points

- Skeletal muscle contractions can be twitches or tetani, isometric or isotonic, eccentric or concentric
- Twitch duration varies 10-fold with muscle fiber type
- Tetanic contraction is achieved by twitch summation
- The isometric length-tension curve is explained by the sliding filament theory
- Isotonic shortening velocity is inversely related to force in Hill’s force-velocity equation
- Hill’s three-element model assume passive and active stresses combine additively
- The series elastance is Hill’s model is probably experimental artifact, but crossbridges themselves are elastic